Analyses of delivery reliability in power systems

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ABSTRACT: This paper describes a newly developed method for analyzing reliability in complex systems such as electrical power systems. The method is based on splitting up Markov models into smaller unit models, thus enabling building large, complex models and analyzing them as a unit but with the original level of details. Traditionally, such analyses have been made using simple models to fit the system into a manageable size or due to restrictions of the applied method. The present method overcomes such limitations, and by incorporating load flow calculations, it is well suited for calculating the load delivery reliability in complex systems for e.g. different load and production levels, different levels and locations of spinning reserves, changing network configurations etc. The application of the method on electrical power system is shown by examples, and applications on other types of complex systems are discussed.

1 INTRODUCTION

The ever increasing complexity of electrical transmission systems makes it difficult to fully understand how the different components interact. Also, the fact that such systems are being more and more pressed, with the aim to increase the utilization, contributes to the increased need for good planning and operational tools.

This paper describes a method for analyzing the reliability of load delivery in electrical power systems. In Chapter 2, the need for a tool as this is discussed from the Transmission System Operator's (TSO's) point of view. Chapter 3 of the paper gives detailed description of the theoretical background of the method. In Chapter 4, application of the method is shown by examples. In Chapter 5 the results are discussed and commented. Finally, in Chapter 6, application of the method on other systems involving flowing media is discussed. Even though this paper is concentrated on the application of the method in electrical power system context, the method, in general, can be applied in any other system involving flowing media.

2 THE TSO POINT OF VIEW

2.1 *Background*

The Transmission System Operator (TSO) is faced with increasing requirements regarding the reliability of load delivery. The cost of not delivering agreed energy can be substantial. The need for powerful analyses tools is therefore obvious. Tools for power flow calculations, dynamic analyses etc. are some of the most important tools for the TSO. To be able to analyze the load delivery reliability, a suitable tool is needed.

Reliability analyses establish the foundation for decisions to be taken regarding operating and planning of the power system, such as [Wang, X. McDonald, J. R. 1993.]:

- Power system planning,
- Assessing different measures,
- Establishing grid philosophy (i.e. operation, future expansions) and grid codes,
- Establishing norms for power delivery reliability,
- Planning of operation, maintenance, and alert.

Analyzing the expected cost of not delivering the agreed amount of energy is also an important issue.

2.2 *The Icelandic transmission system*

Landsnet (Icegrid) is the Icelandic TSO. Until recently, the reliability analyses have been based on the well known "minimal cut set approach" [Høyland; A. Rausand, M. 1994; Billinton, R. Allan, R.N. 1996; Endrenyi, J. 1978.]. This method is very effective when analyzing radial systems, but may reach it's limitations in analyses on meshed grids. This method does not consider load flow and the meaning of spinning reserves when calculating the reliability.

The method, described in Chapter 3 of this paper, uses Markov models composed of unit models. It can easily handle extensive number of reliability states and therefore is well suited for analyzing meshed grids. The method also takes into account the amount and location of spinning reserves.

Although the Icelandic transmission system is fairly small, it is a meshed grid and relatively complicated, with the strongest part (220 kV grid) in the SW-part of Iceland and a 132 kV ring-connection around the island, see figure 1.

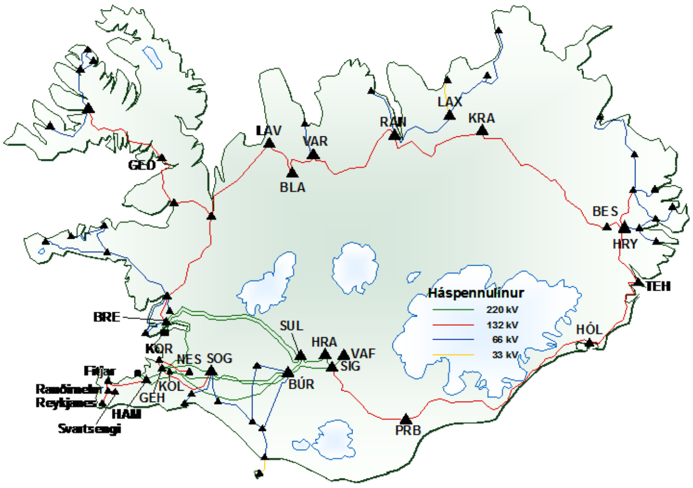

Figure 1. The Icelandic Power Transmission System, as of January 2007.

It is of utmost importance for the TSO to be able to perform detailed and accurate reliability analyses; for the daily operation as well as for future planning (comparison of reinforcement alternatives etc.). By including power flow considerations in the calculations, the operator is able to plan where to locate the spinning reserves to maximize the load delivery reliability. Reliability analyses are also important for system planning, for analyzing different alternatives for network reinforcement's etc.

3 THEORETICAL BASIS

3.1 *Development history*

As responsible for electrical power supply Yngve Aabø experienced the need for new reliability calculation tools. This need results in a master thesis proposal which was carried out by Arne Brufladt Svendsen [Svendsen, A. B. 2002.]. As thesis advisor Tørris Digernes discovered a method suitable for building large Markov / Kolmogorov models. A central clue in this calculation was the Kronecker matrix operators, (Sasty 1999 p. 99). The method was tested by Arne Brufladt Svendsen in his thesis and found to be very efficient for reliability analysis of power systems. Since then various projects have been carried out to make the method more efficient and complete. In 2004 Tørris Digernes MathConsult received a grant from Inovasjon Norge to develop a complete theory for analysis of power systems. And later the same year a cooperation project including Landsnet, Troll Power and MathConsult was established. This project focused on developing efficient methods for reliability analysis of the Icelandic power transmission system. Also this project was partly financed by Inovasjon Norge. The project results in a computer program called PROMAPS (Probability Methods Applied to Power Systems). Much of the theoretical development has been inspired by the excellent textbook of Endrenyi (1978).

The method concerning building of large Markov / Kolmogorov models based on unit models is presented in the comprehensive textbook of Rausand, M and Høyland, A (2004) with reference to the master thesis of Svendsen, A. B. 2004.

3.2 *Calculation principle*

A typical calculation sequence in PROMAPS includes the following main functions:

1. Preparation of input data on text files or as numerical data
2. Creation of the power system model
3. Creation of the unit models
4. Creation of the branch models
5. Calculation of branch reliability
6. Selection of relevant branch reliability states
7. Calculation of maximum power delivery capacity
8. Calculation of power shortage for a given power demand profile
9. Calculation of expected power shortage
10. Separation of the reliability state space in states capable to deliver demanded power and states not capable
11. Calculation of aggregated power delivery models
12. Calculation of reliability data concerning probability, mean visiting duration and visiting frequency for functioning and failed reliability states.
13. Post calculation of various auxiliary variables including economical data.

Important functions among these are further described in the following.

3.3 *Selection of relevant reliability states*

The functioning situation for a grid is systematically partitioned into grid reliability states. Grid reliability state 1 is the situation where all branches are functioning. The other grid reliability states represent situations with failed branches. If the branches have two reliability states {Functioning, Failed}, then the number of system reliability states is 2^n where n is the number of branches. This number is growing very fast with the number of branches. To save computer memory and calculation time this number is reduced to relevant grid reliability states. The reduction is partly based on a maximum number of expected simultaneous failed branches and partly on a functioning reliability limit.

3.4 *Calculation of maximum power supply*

The maximum power supply is calculated by an object function representing the power delivery profit. Maximizing this object function subject to a power system model gives the maximum power delivery capacity. A typical object function is

$$J = -\mathbf{C}_G\mathbf{P} - \mathbf{P}^T\Phi\mathbf{C}_D\mathbf{P} + \mathbf{C}_L\mathbf{P} \tag{1}$$

where J is the object function value [E/s], **P** the branch power vector [W], $\mathbf{C}_G$ a line vector containing specific power generation cost [E/J], Φ a diagonal matrix representing power loss [1/W], $\mathbf{C}_D$ a diagonal matrix containing specific power transmission cost [E/J], and CL a line vector containing specific power delivery price [E/J]. The symbol E is the economical unit.

A more general object function is

$$J = \mathbf{P}^T\mathbf{C}_Q\mathbf{P} + \mathbf{C}_L\mathbf{P} \tag{2}$$

where $\mathbf{C}_Q$ represents quadratic cost terms [E/J] and $\mathbf{C}_L$ linear cost terms with appropriate sign. In this case the power delivery price may be quadratic. Quadratic terms can be used to distribute limited power to several users. Linear prices allocate all power to the user that maximizes the object function. Various strategies for public utility can also be included in this general object function by using quadratic terms in the delivery price. Nonlinear object functions can also be used, but the calculation time may then be increased considerably.

Formally the maximum power delivery capacity can be calculated by the following optimization

$$\hat{\mathbf{P}} = \arg\max_{\mathbf{P}}\left(J / \Pi\right) \tag{3}$$

where Π is the actual power system model.

A simple model describing the grid is

$$\Pi : \begin{cases} \mathbf{MP} = \mathbf{0} \\ \mathbf{P}_{\min} < \mathbf{P} < \mathbf{P}_{\max} \end{cases} \tag{4}$$

where **M** is the grid configuration matrix or reduced incidence matrix, $\mathbf{P}_{\min}$ is the minimum branch capacity and $\mathbf{P}_{\max}$ is the maximum branch capacity.

More detailed models based on current and voltage calculations can also be used. Control variables in these models are generator and transformer settings. However, the simple model defined by (4) is in many cases acceptable.

Failed branches can be modelled by setting $P_{\min} = P_{\max} = 0$ for the actual branches or by removing the branch from the configuration matrix.

The maximum power delivery calculation is in principle similar to electrical short circuit calculations. However, branch capacity limits are not used in short circuit calculations.

3.5 *Calculation of power shortage*

The power shortage is calculated for all relevant reliability states by

$$\mathbf{P}_s = \begin{cases} \mathbf{P}_D - \hat{\mathbf{P}} \ \text{ if } \ \mathbf{P}_D > \hat{\mathbf{P}} \\ \mathbf{0} \ \text{ otherwise} \end{cases} \tag{5}$$

where $\mathbf{P}_s$ is the power shortage and $\mathbf{P}_D$ is demanded power.

Expected power shortage is calculated by multiplying the power shortage for each reliability state with the belonging probability.

3.6 *Basic Reliability theory*

3.6.1 *Markov / Kolmogorov models*

The Markov / Kolmogorov model is a general method to describe the probability that a system is in a state defined by the reliability state vector $\boldsymbol{\xi}$. The model has the following form [Endrenyi 1978].

$$\dot{\mathbf{p}} = \mathbf{A}\mathbf{p} \tag{6}$$

where **p** is the probability vector describing the probability to stay in the reliability states ξ, $\dot{\mathbf{p}}$ is the rate of change of **p** [1/year] and **A** is the transition rate matrix [1/year]. An additional requirement to the model is $\sum \mathbf{p} = 1$.

Dynamic solution of the differential equation (6) requires a starting value for the probability vector **p**. The stationary solution is independent of the starting value.

Two important variables concerning reliability analysis in addition to the probability are the mean visiting durations and visiting frequencies for the different reliability states. These variables are defined by [Endrenyi 1978]:

$$\boldsymbol{\theta} = -diag(\mathbf{A})^{-1} \tag{7}$$

$$\mathbf{v} = diag(\boldsymbol{\theta})^{-1}\mathbf{p} = -diag(\mathbf{A})\mathbf{p} \tag{8}$$

where $\boldsymbol{\theta}$ is mean visiting duration or mean duration vector [year] and $\mathbf{v}$ is the visiting frequency vector [1/year].

The mean duration is the time spent in a reliability state at a visit and the visiting frequency is the number of visits at a reliability state each year.

3.6.2 *Model composition*

Assume a system consisting of 2 components with the component reliability models

$$\dot{\mathbf{p}}_\alpha = \mathbf{A}_\alpha \mathbf{p}_\alpha$$
$$\dot{\mathbf{p}}_\beta = \mathbf{A}_\beta \mathbf{p}_\beta \tag{9}$$

Assume further that the system probability is given by combining the component reliability states. This probability can be expressed as

$$\mathbf{p}_s = \mathbf{p}_\alpha \otimes \mathbf{p}_\beta \tag{10}$$

where $\otimes$ is the Kronecker product [Sasty, S. 1999 p.99]

Time derivation of this expression gives

$$\dot{\mathbf{p}}_s = \dot{\mathbf{p}}_\alpha \otimes \mathbf{p}_\beta + \mathbf{p}_\alpha \otimes \dot{\mathbf{p}}_\beta$$

Inserting the component equations (9) and rearranging gives further [Sasty, S. 1999 p.99]

$$\dot{\mathbf{p}}_s = \mathbf{A}_s \mathbf{p}_s \tag{11}$$

$$\mathbf{A}_s = \mathbf{A}_\alpha \otimes \mathbf{I}_\beta + \mathbf{I}_\alpha \otimes \mathbf{A}_\beta \tag{12}$$

where $\mathbf{A}_s$ is the composite transition rate matrix, $\mathbf{I}_\alpha$ and $\mathbf{I}_\beta$ are identity matrixes with dimensions corresponding to the probability vectors.

Note that equation (11) is in standard Markov / Kolmogorov form. Mean visiting durations and visiting frequencies are therefore also available from the transition rate matrix $\mathbf{A}_s$ for the composite system model.

This principle can be used recursively to compose large dimensional models based on simple unit models.

3.6.3 *Model aggregation*

High dimension probability vectors are difficult to interpret. More insight is obtained by aggregating the probability vector. Typical aggregated reliability states are {The system is functioning, The system has failed}.

Aggregation of a probability vector can be defined by

$$\mathbf{p}_a = \mathbf{D}\mathbf{p}_s \tag{13}$$

where $\mathbf{p}_a$ is the aggregated probability vector, $\mathbf{p}_s$ is the un-aggregated system probability vector and $\mathbf{D}$ is the aggregation matrix. An additional requirement is $\sum \mathbf{p}_a = 1$. This requirement is satisfied if the column sums of $\mathbf{D}$ are 1.

Also an aggregated transition rate matrix can be calculated according to a function of the type

$$\mathbf{A}_a = \mathbf{f}_a(\mathbf{D}, \mathbf{A}_s, \mathbf{p}_s) \tag{14}$$

It is possible to show that the aggregated variables satisfy the standard Markov / Kolmogorov equation

$$\dot{\mathbf{p}}_a = \mathbf{A}_a \mathbf{p}_a \tag{15}$$

The matrix $\mathbf{A}_a$ contains therefore the necessary information to calculate the aggregated mean visiting durations and visiting frequencies.

3.7 *Creation of branch models*

A branch may consist of the following main components:
- Generators or power import branches,
- Transmission lines,
- Transformers,
- Load or export branches.

In addition these components may contain a lot of protection and reconnection functions. For a branch to be in functioning state a necessary amount of these components have to be functioning. A complete reliability model for each branch is build by model composition taking into account all relevant components. A typical branch model may have from 1000 to 20.000 reliability states. Finally these models are reduced to normally the two reliability states {Functioning, Failed} by model aggregation.

3.8 *Delivery shortage cost*

The power shortage cost is obtained by multiplying the expected power shortage and a defined power shortage price for each load point.

Also a long term profit can be analyzed by a dynamic cost flow model that includes capital accumulation, investments, income and various cost flows.

3.9 *Realization*

The theory is realized in a windows based computer program. The program has available a library of unit models for different power grid components and new models can easily be included. Input data can be defined on text files or entered directly from the screen. The main window contains a single line diagram with possibilities to operate the grid, change parameter values and inspect calculation results. All commands are available as pull down menus or as context menus accessible by right clicking the mouse. The calculation results can be presented as curves, bars and lists.

4 APPLICATION OF THE METHOD

4.1 *The Icelandic transmission system*

In the development project the Eastern and Western parts of the Icelandic transmission system are modeled. Typical size of these grids is:

Grid	Nodes	Branches	Generators	Loads
Eastern	9	10	5	6
Western	10	17	17	6

Table 1 Typical size of the two main parts of the Icelandic Transmission system

Especial the Western grid is highly meshed.
Experiences from these projects confirm that important factors that influence the power delivery reliability are:
- Component reliability,
- Branch capacity,
- Normal power production,
- Spinning reserve power production,
- Location of the power production,
- Grid operation strategy,
- Load flow,
- Load priority.

The experiences also confirm that decision of the reliability condition in a meshed power system grid is highly difficult without analysis tools. Tools of this type give in addition new insight into the reliability behavior of complex power systems.

Simulation experience from these models indicates that considerably larger systems can be analyzed. However, to simplify the presentation a low size demonstration model will be used in the next section.

4.2 *Results from a demonstration model*

4.2.1 *Grid structure*

The demonstration model is based on the simplified grid shown in figure 2.

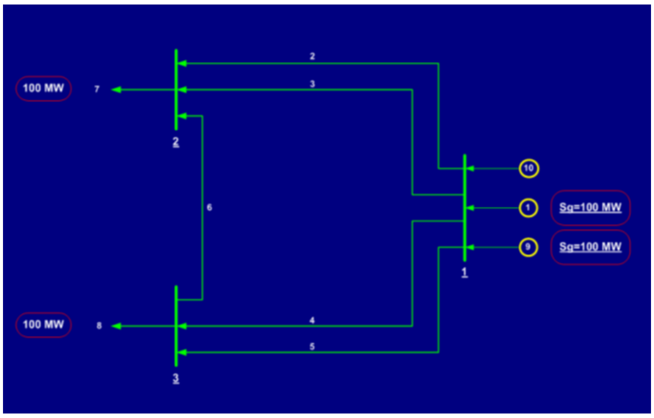

Figure 2. Demonstration grid

The branch types are:

Branch	Branch type	Capacity [MW]
1	Normal production power generator	100 (200)
9	Normal production power generator	100
10	Spinning reserve power generator	50
2	Transmission line	100
3	Transmission line	100
4	Transmission line	100
5	Transmission line	100
6	Transmission line	50
7	Load	100
8	Load	100

Table 2 Branch types

The spinning reserve power generation is only supplying power if the normal production power generators are not able to deliver demanded power.
This is obtained by giving the spinning reserve generators higher production cost than the normal production generators in the object function.

4.2.2 *Analysis cases*

The following three situations based on the demonstration grid are analyzed:

Case 1:

Branch	Branch type	Capacity
1	Power generator	200 MW
9	Power generator	0 MW
10	Spinning reserve power generator	0 MW

Table 3

Case 2:

Branch	Branch type	Capacity
1	Power generator	100 MW
9	Power generator	100 MW
10	Spinning reserve power generator	0 MW

Table 4

Case 3:

Branch	Branch type	Capacity
1	Power generator	100 MW
9	Power generator	100 MW
10	Spinning reserve power generator	50 MW

Table 5

A selection of variables calculated by the computer program PROMAPS are presented in the following sections.

4.2.3 *Power flow*

The power flow in normal situation for case 3 is show in figure 3. Dark blue color represents the actual power flow in each branch. Light blue color represents the branch capacity. Positive and negative values describe the power flow direction. Positive values indicate that the power flow is in the arrow direction in the single line diagram.

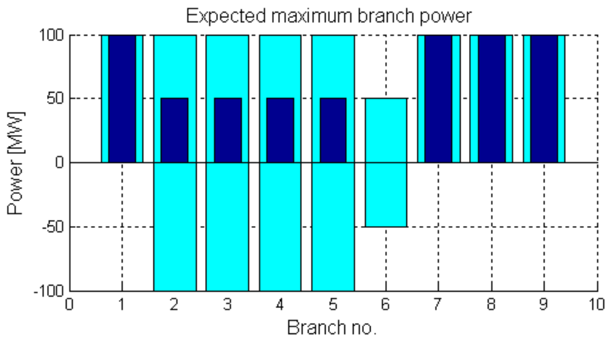

Figure 3. Load flow in normal situation

Figure 3 indicates that the production units, branch 1 and 9 are supplying power to load branch 7 and 8 through the parallel lines, branch 2 and 3, and the parallel lines, branch 4 and 5. At normal situation there is no power in branch 6. The spinning reserve is also at this situation zero, and shall only generate power if the normal production generators are not able to supply demanded power.

In figure 4 transmission line branch 2 is faulted.
In this case the power flow has increased in the functioning transmission lines. The extra power flow in lines 4 and 5 is transferred to branch 6, which is now also supplying power to load branch 7.

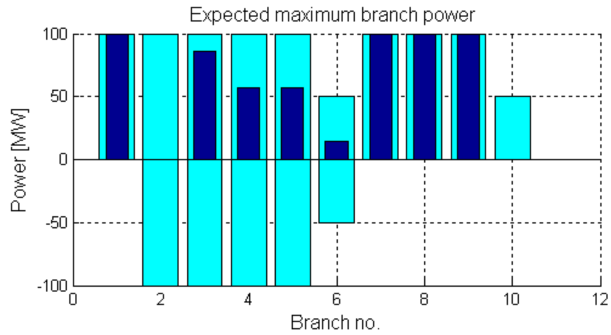

Figure 4. Load flow in fault situation

4.2.4 *Delivery reliability*

The delivery reliability for branch 8 is shown in the next figure for the tree demonstration cases.

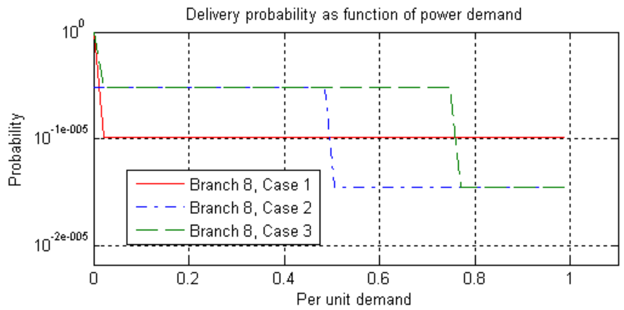

Figure 5. Delivery probability

The delivery reliability for load branch 7 is almost similar to load branch 8.

The delivery probability is very near 1 for a normal functioning grid, as shown in the figure. It is therefore normally minimal information in the curves. However, these curves will indicate if unacceptable components are used.

4.2.5 *Mean visiting duration in functioning states*

The mean visiting durations in functioning state for load point 8 are shown in the following figure for the actual analysis cases.

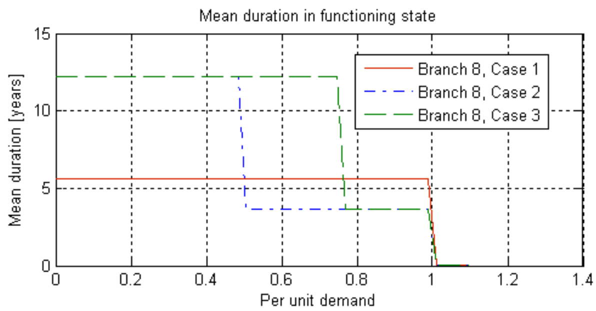

Figure 6. Mean duration in functioning state for load point 8.

In all these simulation cases the mean duration in functioning state for load point 7 is identical to load point 8.

The simulations results indicate that:
Case 1: The mean duration in functioning state is approximately 5.6 years from 0 to maximum load capacity.

Case 2: The mean duration in functioning state is approximately 12.2 years from 0 to 50% load and then approximately 3.6 year up to 100% load.

Case 3: For this case is the range having mean duration time 12.2 years increased to 75% load.

These simulations show that the mean duration in functioning state or mean time to failure for the load delivery varies in steps based on the power generation capacity. In situations where capacity limits is reached for one of the branches in a parallel structure, the reliability of the parallel branches behaves like serial structures. Including spinning reserve in the system contributes to increase the delivery reliability.

In these simulation cases focus has been on the production units and spinning reserve. The same principle applies to all the branches in a meshed system. Assume two parallel lines supplying one load branch can deliver 80 % each of a power demand. Then the redundancy only applies up to 80 %. Above 80 % the parallel power lines act as a series structure, which means that both lines have to be in operation for transmission of power at this level. The grid will at this load change reliability behavior from a parallel structure to a serial structure. This type of change in reliability behavior makes analysis of large meshed grids very complex.

4.2.6 *Power shortage*

The power shortage for the three simulation cases is shown in the following figure.

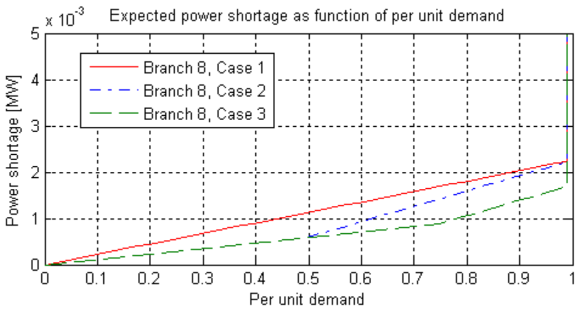

Figure 7. Expected power shortage as function of per unit demand

This figure indicates, as assumed, that the expected power shortage is slightly increasing until maximum load capacity and then is increasing very fast. Case 3 with spinning reserve has the lowest power shortage cost.

4.2.7 *Consequences of disconnecting a branch*

Assume that branch 2 is disconnected due to a maintenance situation. The mean visit duration in functioning state changes in this situation as shown in the following diagram.

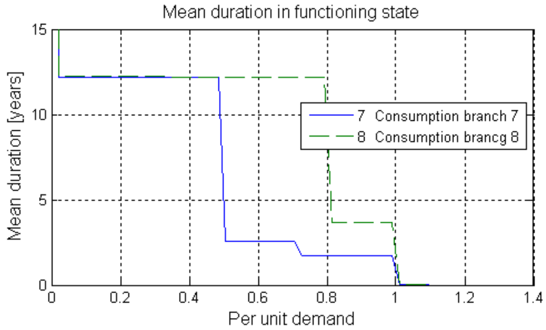

Figure 8. Mean duration in functioning state with branch 2 disconnected

Expected power flow in this situation is shown in the following figure.

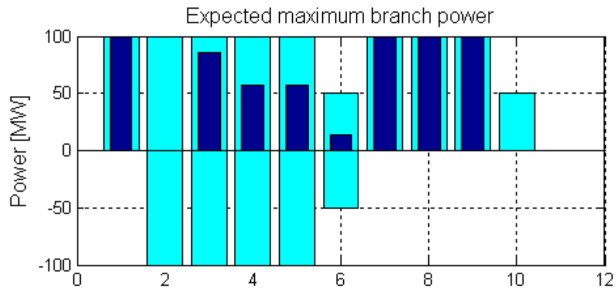

Figure 9. Expected maximum branch power

This figure indicates a possible overload in branches 1 and 9.

4.2.8 *Other variables*

Other central variables that can be calculated are:
- Critical situations that my cause grid collapse,
- Expected power shortage cost,
- Power shortage probability,
- Mean visit duration in power shortage states,
- Visiting frequency,
- Per unit power shortage,
- Power unavailability,
- Failure causes,
- Main contributions to power shortages.

4.3 *Calculation capacity*

The program is tested for grids having up to 60 branches with typical calculation time around 25s for the most complicated analysis functions. Several improvements to make the program more efficient seem to be possible.

4.4 *Expected power shortage as a design and maintenance specification*

The diagram in figure 10 shows per unit power shortage as function of per unit power demand for a specific load point. The per unit values are normalized by the maximum load capacity. A precise reliability design requirement or maintenance requirement is stated by defining the maximum load capacity and the acceptable per unit power shortage curve for each load point. Expected power shortage curves are calculated in PROMAPS.

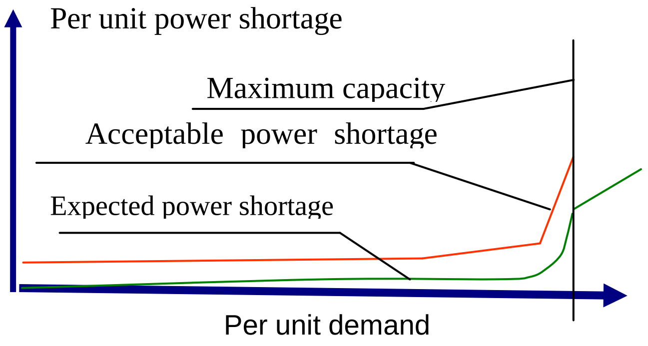

Figure 10. Expected power shortage

5 DISCUSSION AND COMMENTS

The presented method is quite new, but available experiences indicate that the method is surprisingly efficient, accurate, fast, and flexible. Large systems with high degree of details can be analyzed. A number of interesting variables can be presented; including economical data. In addition, the method seems, with some modifications, to be efficient for analysis of systems having nonlinear reliability models.

The simulation cases presented in this paper demonstrate the influence of spinning reserve power production. The results verify that the reliability limits in a power system depend on the amount of spinning reserve, the number of production units, location of the production units, the load flow and load demand and the power capacity. In addition the load priority will also affect the reliability results. The lowest prioritized load is disconnected if the power demand is higher than the production capacity. Such disconnections have complex influences on the delivery reliability. The program can be used to analyze various strategies for disconnection.

Since a power system is dynamic, variables such as power demand, power production, load flow, and short circuit levels will continuously change. The same changes also apply for the power delivery reliability results at each load branch. It is important to be aware of this fact, when operating a power system. Different action and changes in a power system will change the reliability of power delivery. Power grid operators and power system planners should be aware of this and make their decisions based on the best information available.

Typical applications of the method are power system concept studies, maintenance planning, operation planning, protection strategy planning, power loss minimization and on line calculation of existing grid reliability.

6 “NON-ELECTRICAL” APPLICATIONS

Typical non-electrical applications are fluid flow systems including oil flow, and general material transport. Presumably also traffic and communication systems can be analyzed by this method. In these cases the power flow model is replaced by the actual system model. The principle based on unit models can also be used in general probability and reliability calculations to build extremely large models.

7 ACKNOWLEDGEMENTS

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